

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 (10p)

- (a) Define the following concepts: *structural induction*, *De Morgan's laws*, and *Herbrand structure*. (3 × 2p)
- (b) What is meant by the notation $\Sigma \models \phi$?
Prove in detail that if $\Sigma \models \phi$ and $\Sigma \models \neg\phi$, then Σ is unsatisfiable. (4p)

Assignment 2 (10p) Prove the following claims using semantic tableaux:

- (a) $\models (A \rightarrow B) \rightarrow ((\neg A \rightarrow \neg C) \wedge (B \rightarrow D) \rightarrow (C \rightarrow D))$.
- (b) $\{\exists x(A(x) \vee B(x) \vee C(x))\} \models \forall x\neg A(x) \rightarrow (\forall y\neg B(y) \rightarrow \exists zC(z))$.

Tableau proofs must contain all intermediary steps !!!

Assignment 3 (10p) Derive a Prenex normal form and a clausal form (i.e., a set of clauses S) for the sentence $\neg(\forall xP(x) \vee \exists y\forall xQ(x,y) \rightarrow \forall x(P(x) \vee \exists yQ(x,y)))$.

Make S as simple as possible. Prove that S is unsatisfiable using resolution.

Assignment 4 (10p) Let us represent natural numbers $0, 1, 2, \dots$ using ground terms $0, s(0), s(s(0)), \dots$ built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x .

- (a) Define a predicate $D(x,y) =$ “number y is twice as big as number x ” using sentences of predicate logic so that your definition covers all natural numbers (represented in the way explained above).
- (b) Provide a counter model, on the basis of which your definition does not entail

$$\forall x\forall y\forall z(D(x,y) \wedge D(y,z) \rightarrow D(x,z)).$$

Assignment 5 (10p)

- (a) Derive for the program `if (x < y) then {z = y - x} else {z = x - y}` the *weakest precondition* starting from the *postcondition* $(z > 0)$. (4p)
- (b) Consider the following program Copy:

```
z = x ; y = 0 ; while (! (z == 0)) {y = y + 1 ; z = z - 1}.
```

Use weakest preconditions and a suitable invariant (6p) to establish

$$\models_p [\text{true}] \text{Copy} [(y == x)].$$