Aalto University School of Science, Department of Information and Computer Science TJ

T-79.5104 Advanced Course in Computational Logic

Examination, March 9, 2012

Assignment 1 (Max. 10p)

(a) Define formally *frame p-morphisms*. What is the fundamental property expected from them?

(b) Given a set of frames **L**, define the *logical consequence* relation $\Sigma \models_{\mathbf{L}} \Upsilon \Longrightarrow P$. Describe the form of respective deduction theorems for modal logic.

Assignment 2 (Max. 10p) Use the *tableau method* to determine whether the following claims hold. Give a counter-model based on the tableau if appropriate. (Symbols P and Q denote atomic propositions below.)

(a) The formula $\Box P \rightarrow \Box\Box P$ is a **K**-consequence (where **K** is the set of all frames) of the global premise $\Box(\Box P \rightarrow P) \rightarrow \Box P$.

(b) There is a model based on a transitive and reflexive frame and a possible world in the model where formulas $\Diamond Q$ and $\Diamond(P \wedge \neg\Box Q)$ are false but $\Box\Diamond\Box\Diamond P$ is true.

Assignment 3 (Max. 10p)

(a) Show that **KD** based on serial frames is the smallest normal modal logic containing the formula $\Diamond\top$.

(b) Consider a Hilbert-style proof system whose axioms are all classical tautologies and all formulas of the forms $\Box P \rightarrow \Box\Box P$ and $\Box P \rightarrow P$ and whose inference rules are Modus Ponens and the necessitation rule. Define when a Hilbert-style proof system is sound and complete for a given modal logic L. Show that the proof system above is complete but not sound for the modal logic **K4** based on transitive frames.

Assignment 4 (Max. 10p)

(a) Define the following concepts in $\mathcal{ALC}$ extended by *inverse roles* using the concept name `Worker` and the role name `supervises`:

1. A manager (a worker who supervises at least one worker)
2. A director general (a manager who supervises only mangers and is not supervised by any worker)

(b) Consider a knowledge base $(\mathcal{T},\mathcal{A})$ having TBox $\mathcal{T} = \{A \sqsubseteq C, B \sqsubseteq C\}$ and ABox $\mathcal{A} = \{a : (\exists r.A \sqcup \exists r.B)\}$ where A, B, and C are concept names, r is a role name, and a is an individual name.

Use the tableau algorithm for $\mathcal{ALC}$ to study whether the KB $(\mathcal{T},\mathcal{A})$ entails that the individual a is an instance of the concept $(\exists r.C)$ and give a counter model if appropriate.

Properties of a relation R:

Reflexive:	$\forall s(sRs)$		Serial:	$\forall s\exists t(sRt)$
Symmetric:	$\forall s\forall t(sRt \rightarrow tRs)$		Euclidean:	$\forall s\forall t\forall u(sRt \wedge sRu \rightarrow tRu)$
Transitive:	$\forall s\forall t\forall u(sRt \wedge tRu \rightarrow sRu)$			

The name of the course, the course code, the date, your name, your student identifier, and your signature must appear on every sheet of your answers.