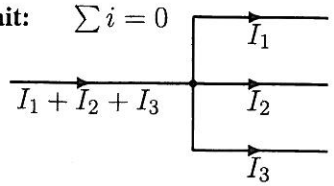
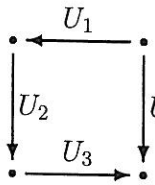
S-55.1100 SÄHKÖTEKNIIKKA JA ELEKTRONIIKKA, Kaavakokoelma (*Kako*) 30.1.2012 © X

Ota *Kako* mukaan välikokeisiin ja tenttiin. Kaikki kaavat eivät ole yleispäteviä. Selvitä itsellesi kirjainten merkitys, ja milloin mitäkin laskutapaa voi käyttää. Älä opettele muita kaavoja ulkoa. Prefiksit: y, z, a, f, p, n, μ, m, 1, k, M, G, T, P, E, Z, Y. Kreikkalaiset: alfa α, beta β, gamma γ Γ, delta δ Δ, epsilon ε, zeta ζ, eta η, theta θ, jota ι, kappa κ, lamda λ Λ, myy μ, nyy ν, ksi ξ, omikron o, pii π Π, rho ρ, sigma σ Σ, tau τ, ypsilon υ, phi ϕ φ, khi χ, psi ψ, omega ω Ω.

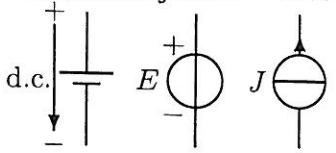
1. Kirchhoffin lait: $\sum i = 0$



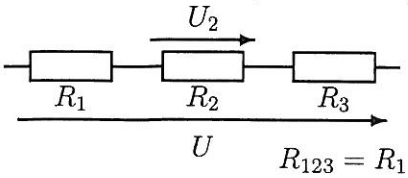


$\sum u = 0$ $\quad U_1 + U_2 + U_3$

2. Lähteet: jännite virta

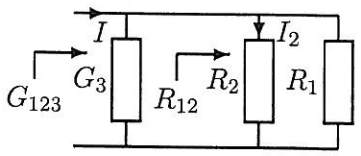


3. Jännitteen jako (sarjassa): $U_2 = \frac{R_2}{R_1+R_2+R_3}U$



$R_{123} = R_1 + R_2 + R_3$

4. Virran jako (rinnankytkennässä): $I_2 = \frac{G_2}{G_1+G_2+G_3}I$



$R_{12} = \frac{R_1R_2}{R_1+R_2} = \frac{1}{G_{12}}$

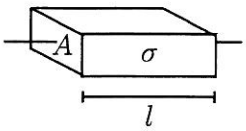
$G_{123} = G_3 + \overbrace{G_2 + G_1}^{G_{12}}$

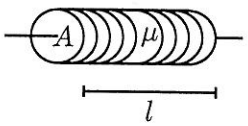
5. Virtauskenttä: $U = El \quad I = JA \quad E = \rho J = \frac{J}{\sigma}$ **6. Resistanssi ja konduktanssi:** $R = \frac{U}{I} = \frac{1}{G}$

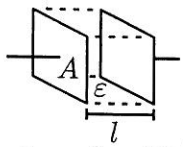
7. Induktanssi: $u = L\frac{\mathrm{d}i}{\mathrm{d}t} \Leftrightarrow i = \frac{1}{L}\int_0^t u\mathrm{d}t + I_{\mathrm{L}0} \quad u = \frac{\mathrm{d}\psi}{\mathrm{d}t} \quad L = \frac{\psi}{I} \quad w_{\mathrm{L}} = \frac{1}{2}Li^2$

8. Kapasitanssi: $i = C\frac{\mathrm{d}u}{\mathrm{d}t} \Leftrightarrow u = \frac{1}{C}\int_0^t i\mathrm{d}t + U_{\mathrm{C}0} \quad i = \frac{\mathrm{d}q}{\mathrm{d}t} \quad C = \frac{Q}{U} \quad w_{\mathrm{C}} = \frac{1}{2}Cu^2$

9. Vastus: $R = \frac{\rho l}{A} \quad G = \frac{1}{R} = \frac{\sigma A}{l}$ **10. Lieriökäämi:** $L = N^2\frac{\mu A}{l}$ **11. Tasokondensaattori:** $C = \frac{\varepsilon A}{l}$







12. Magneettikenttä: $H = \frac{U_{\mathrm{m}}}{l} \quad U_{\mathrm{m}} = NI \quad B = \mu H \quad \psi = N\phi \quad \phi = BA \quad \mu = \mu_{\mathrm{r}}\mu_0 \quad \mu_0 = 4\pi \cdot 10^{-7}\ \frac{\mathrm{H}}{\mathrm{m}}$

13. Sähkökenttä: $E = \frac{U}{l} \quad D = \varepsilon E \quad \psi = Q = DA \quad \varepsilon = \varepsilon_{\mathrm{r}}\varepsilon_0 \quad \varepsilon_0 = 8{,}854 \cdot 10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}} = \frac{1}{c_0^2\mu_0} \approx \frac{1}{36\pi}\ \frac{\mathrm{nF}}{\mathrm{m}}$

14. Muutosilmiöt: $\left.\begin{array}{l} u_{\mathrm{C}}(0^-) = u_{\mathrm{C}}(0^+) \\ i_{\mathrm{L}}(0^-) = i_{\mathrm{L}}(0^+) \end{array}\right\}$ **Yritteet:** $\left.\begin{array}{l} i(t) \\ u(t) \end{array}\right\} = \left\{\begin{array}{ll} i(\infty) + [i(0) - i(\infty)]\,\mathrm{e}^{-\frac{t}{\tau}} \\ B \quad + \quad A\,\mathrm{e}^{-\frac{t}{\tau}} \end{array}\right. \quad \tau = \left\{\begin{array}{l} RC \\ L/R \end{array}\right.$

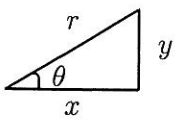
15. Tehollisarvo: $U = \sqrt{\frac{1}{T}\int_0^T u^2\mathrm{d}t} = \sqrt{\sum U_i^2}$ **16. Keskiarvot:** $U_{\mathrm{av}} = \frac{1}{T}\int_0^T u\,\mathrm{d}t \quad U_{\mathrm{r}} = \frac{1}{T}\int_0^T |u|\mathrm{d}t$

17. Sinimuotoinen virta tai jännite: $u(t) = \hat{u}\sin(\omega t + \phi) = \mathrm{Im}[\hat{u}\mathrm{e}^{\mathrm{j}(\omega t+\phi)}] \quad \hat{u} = \sqrt{2}\,|\underline{U}| = \frac{\pi}{2}\,U_{\mathrm{r}} \quad \hat{i} = \sqrt{2}\,|\underline{I}|$

18. Osoitinlaskenta (i, u)**:** $i(t) = \hat{i}\sin(\omega t + \phi) \Leftrightarrow \underline{I} = \frac{\hat{i}}{\sqrt{2}}\angle\phi$ **19. Kulmataajuus:** $\omega = 2\pi f = 2\pi/T$

20. ***Kompleksiluvut:*** $\left\{\begin{array}{ll} z_1 = 4 + \mathrm{j}2 = \sqrt{20}\angle 26{,}6° = \sqrt{20}\,\mathrm{e}^{\mathrm{j}\frac{\pi\ \mathrm{rad}}{6{,}776}} & (26{,}565° = \alpha) \\ z_2 = 3 + \mathrm{j}1 = \sqrt{10}\angle 18{,}4° = \sqrt{10}\,\mathrm{e}^{\mathrm{j}\frac{\pi\ \mathrm{rad}}{9{,}764}} & (18{,}435° = \beta) \end{array}\right. \quad \mathrm{j} = \pm\sqrt{-1}$

21. ***Koordinaatistomuunnos:*** $\left\{\begin{array}{l} x = |r|\cos\theta \\ y = |r|\sin\theta \end{array}\right. \quad \left\{\begin{array}{l} r = \sqrt{x^2 + y^2} = |z| \\ \theta = \arctan(y/x) \end{array}\right.$

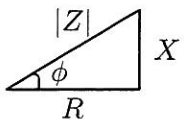


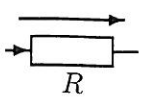
22. ***Summamuoto:*** $\left\{\begin{array}{l} z_1 + z_2 = (4+3) + \mathrm{j}(2+1) = 7 + \mathrm{j}3 \quad z_1z_2 = (4\cdot 3 - 2\cdot 1) + \mathrm{j}(4\cdot 1 + 2\cdot 3) = 10 + \mathrm{j}10 \\ z_1 - z_2 = (4-3) + \mathrm{j}(2-1) = 1 + \mathrm{j} \quad \frac{z_1}{z_2} = \frac{z_1z_2^*}{z_2z_2^*} = \frac{(4\cdot 3+2\cdot 1)+\mathrm{j}(2\cdot 3-4\cdot 1)}{3^2+1^2} = 1{,}4 + \mathrm{j}0{,}2 \end{array}\right.$

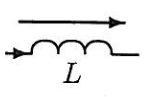
23. ***Kulmamuoto, kerto:*** $z_1z_2 = \sqrt{20}\sqrt{10}\angle(\alpha+\beta) = \sqrt{200}\angle 45°$ ***Jako:*** $\frac{z_1}{z_2} = \frac{\sqrt{20}}{\sqrt{10}}\angle(\alpha-\beta) = \sqrt{2}\angle 8{,}13°$

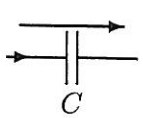
24. ***Liittoluku ja itseisarvo:*** $z = x + \mathrm{j}y = |z|\angle\theta \Leftrightarrow z^* = x - \mathrm{j}y = |z|\angle -\theta \quad zz^* = |z|^2 \quad |z| = \sqrt{x^2+y^2}$

25. Impedanssi: $U = ZI \quad Z = R + \mathrm{j}X$ **26. Admittanssi:** $I = YU \quad Y = G + \mathrm{j}B = \frac{1}{Z} = \frac{R-\mathrm{j}X}{R^2+X^2}$



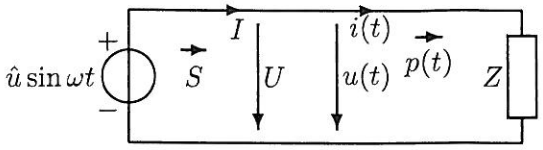


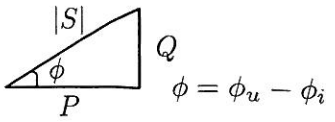




27. Vastus: $Z_{\mathrm{R}} = R$ **28. Kela:** $Z_{\mathrm{L}} = \mathrm{j}\omega L$ **29. Kondensaattori:** $Z_{\mathrm{C}} = \frac{1}{\mathrm{j}\omega C} = -\mathrm{j}\frac{1}{\omega C}$

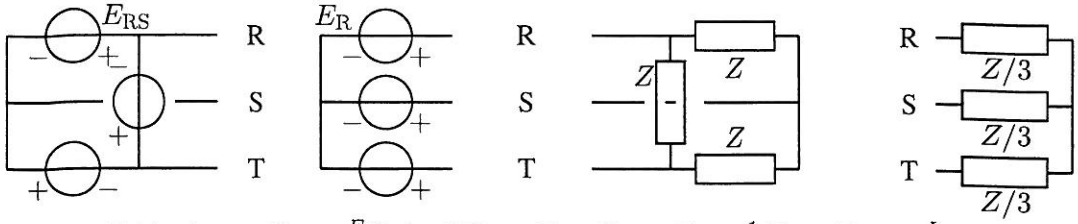
30. Teho: $S = UI^* = P + \mathrm{j}Q \Rightarrow P = \frac{1}{T}\int_0^T p(t)\mathrm{d}t = |U||I|\cos\phi \quad Q = |U||I|\sin\phi \quad p(t) = u(t)i(t)$





$\phi = \phi_u - \phi_i$

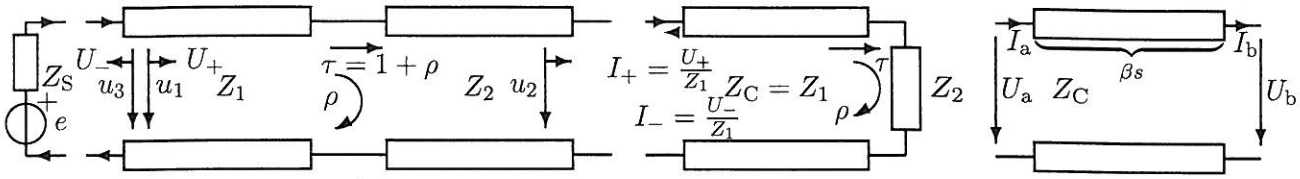
31. Kolmivaihejärjestelmä (U, I)**:** $U_\mathrm{R} = U\angle 0° \quad U_\mathrm{S} = U\angle -120° \quad U_\mathrm{T} = U\angle -240° = U\angle +120°$



32. 1-vaiheinen sijaiskytkentä: $E_\mathrm{R} = \frac{E_\mathrm{RS}}{\sqrt{3}}\angle -30° \quad Z = Z_\Delta \quad Z_\mathrm{Y} = \frac{1}{3}Z \quad I_\mathrm{RS} = \frac{I_\mathrm{R}}{\sqrt{3}}\angle 30°$

33. Ominaisimpedanssi ja aallon nopeus: $Z_\mathrm{C} = \frac{U_+}{I_+} = \frac{U_-}{I_-} = \sqrt{\frac{l}{c}} \quad v = \frac{s}{\Delta t} = \frac{c_0}{\sqrt{\epsilon_\mathrm{r}\mu_\mathrm{r}}} = \frac{1}{\sqrt{lc}} \quad c_0 \approx 3 \cdot 10^8$ m/s

34. Heijastus- ja läpäisykerroin: $\rho = \frac{u_3}{u_1} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad \tau = \frac{u_2}{u_1} = 1 + \rho = \frac{2Z_2}{Z_2 + Z_1} \quad u_1(0) = U_+ = Z_1 \frac{e}{Z_\mathrm{S} + Z_1}$



35. Siirtojohtoyhtälöt (sini): $\begin{cases} U_\mathrm{a} = U_\mathrm{b}\cos\beta s + \mathrm{j}Z_\mathrm{C}I_\mathrm{b}\sin\beta s \\ I_\mathrm{a} = \mathrm{j}\frac{U_\mathrm{b}}{Z_\mathrm{C}}\sin\beta s + I_\mathrm{b}\cos\beta s \end{cases} \quad \beta = \frac{\omega}{v} = \frac{2\pi}{\lambda} = \frac{360°}{\lambda} \quad v = \lambda f$ **Välikoeraja:**

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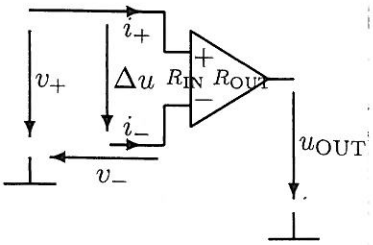
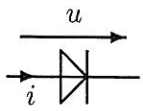
36. Operaatiovahvistin: $\begin{cases} v_+ = v_- \Rightarrow \Delta u = 0 \\ i_+ = 0 \\ i_- = 0 \end{cases} \quad \begin{cases} A_u = \frac{u_\mathrm{OUT}}{\Delta u} = \infty \\ R_\mathrm{IN} = \infty \\ R_\mathrm{OUT} = 0 \end{cases}$

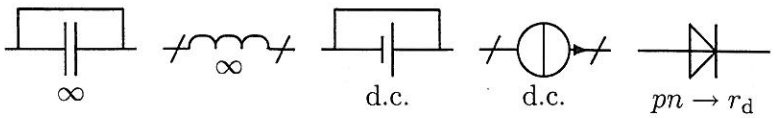


37. Diodi: $\begin{cases} i = I_\mathrm{S}(\mathrm{e}^{\frac{u}{nU_T}} - 1) \\ u = nU_T \ln\frac{i + I_\mathrm{S}}{I_\mathrm{S}} \end{cases} \quad i \approx \begin{cases} I_\mathrm{S}\left(\mathrm{e}^{\frac{U}{nU_T}}\right) & (U >> nU_T) \\ I_\mathrm{S}(-1) & (U << 0) \end{cases} \quad U_T = \frac{\mathrm{k}T}{\mathrm{q}} \approx 25\ \mathrm{mV} \quad n \approx 2$

$\mathrm{q} = 1{,}602 \cdot 10^{-19}$ As $\qquad \mathrm{k} = 1{,}381 \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}} \qquad r_\mathrm{d} = \frac{1}{\frac{\partial i}{\partial u}\big|_U} = \frac{nU_T}{I}$

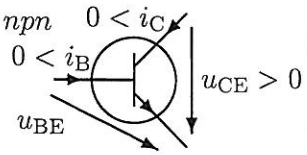


38. Piensignaalianalyysi:

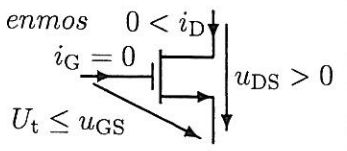


39. Transistori (BJT): $i_\mathrm{C} = \beta i_\mathrm{B} = \alpha i_\mathrm{E}$ kun $u_\mathrm{CE} \geq 0{,}3$ V $\quad u_\mathrm{BE} \approx 0{,}7$ V $\quad n \approx 1$

$r_\pi = \frac{nU_T}{I_\mathrm{B}} = \frac{u_\mathrm{be}}{i_\mathrm{b}} \qquad g_\mathrm{m} = \frac{\beta}{r_\pi} \qquad i_\mathrm{c} = \beta i_\mathrm{b} = g_\mathrm{m}u_\mathrm{be} \qquad r_\mathrm{o} = \frac{U_\mathrm{A}}{I_\mathrm{C}}$



40. FET: $\begin{cases} \mathrm{CUT}: & u_\mathrm{GS} \leq U_\mathrm{t} \Rightarrow i_\mathrm{D} = 0 \\ \mathrm{OHM}: & u_\mathrm{DS}^2 \approx 0: \quad i_\mathrm{D} = i_\mathrm{S} \approx \overbrace{2K(u_\mathrm{GS} - U_\mathrm{t})}^{\approx 1/r_\mathrm{DS}} u_\mathrm{DS} \\ \mathrm{TRI}: & u_\mathrm{DS} \leq u_\mathrm{GS} - U_\mathrm{t}: \quad i_\mathrm{D} = i_\mathrm{S} = K[2(u_\mathrm{GS} - U_\mathrm{t})u_\mathrm{DS} - u_\mathrm{DS}^2] \\ \mathrm{SAT}: & u_\mathrm{DS} \geq u_\mathrm{GS} - U_\mathrm{t}: \quad i_\mathrm{D} = i_\mathrm{S} = K(u_\mathrm{GS} - U_\mathrm{t})^2 \end{cases}$



$i_\mathrm{d} = g_\mathrm{m}u_\mathrm{gs} \qquad g_\mathrm{m} = 2K(U_\mathrm{GS} - U_\mathrm{t}) = 2\sqrt{KI_\mathrm{D}} \qquad r_\mathrm{o} = \frac{U_\mathrm{A}}{I_\mathrm{D}}$

41. Tehoelektroniikka: $\Delta Q = C\Delta u = I\Delta t \qquad \Delta\psi = L\Delta i = U\Delta t \qquad \Delta T = T_\mathrm{A} - T_\mathrm{B} = \theta_\mathrm{AB}P = R_\mathrm{TH}P$